today:

homework 3 due (6.2.16, 6.2.22, 6.2.60, 6.3.20, 6.3.28, 6.3.46b)
§ 7.1 - integration by parts
§ 7.2 - trig integrals
mslc - trig review workshop @ 12:30 and 3:30 in CH 042

monday:

webwork extra credit project 1 due @ 6:00 am

tuesday:

homework 4 due (6.4.10, 6.4.16, 7.1.28, 7.1.56, 7.2.44, 7.2.66)
mslc: integration techniques workshop @ 12:30 and 3:30 in CH 042
quiz: §§ 6.4, 7.1
§ 7.3 - trig substitution
review

wednesday:

webwork 4 due @ 11:55 pm
mslc: webwork workshop @ 12:30, 1:30, 2:30, 3:30, 4:30 in SEL 040
mslc: integration techniques workshop @ 1:30 and 3:30 in CH 042
mslc: midterm review @ 7:30 pm in HI 131

thursday, 29 october:

midterm: §§ 6.2-6.4, 7.1-7.3
§ 7.4 - partial fractions

tuesday, 3 november:

homework 5 due (7.3.8, 7.3.22, 7.3.40, 7.4.20, 7.4.48, 7.4.50)

integration by parts

integration by parts

The product rule states

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\,g(x)\right) = f(x)\,g'(x) + f'(x)\,g(x)$$

rearranging terms, we have

$$f(x)\,g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\,g(x)\right) - f'(x)\,g(x)$$

integration by parts

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$$f(x)\,g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\,g(x)\right) - f'(x)\,g(x)$$

integrating,

$$\int f(x)\,g'(x)\,\mathrm{d}x = \int \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\,g(x)\right)\,\mathrm{d}x - \int f'(x)\,g(x)\,\mathrm{d}x$$

integration by parts

integrating,

$$\int f(x)\,g'(x)\,\mathrm{d}x = \int \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\,g(x)\right)\,\mathrm{d}x - \int f'(x)\,g(x)\,\mathrm{d}x$$

so by the fundamental theorem of calculus,

$$\int f(x)\,g'(x)\,\mathrm{d}x = f(x)\,g(x) - \int g(x)\,f'(x)\,\mathrm{d}x$$

integration by parts

so by the fundamental theorem of calculus,

$$\int f(x)\,g'(x)\,\mathrm{d}x = f(x)\,g(x) - \int g(x)\,f'(x)\,\mathrm{d}x$$

letting $u = f(x)$ and $v = g(x)$, we conclude

$$\int u\,\mathrm{d}v = u\,v - \int v\,\mathrm{d}u$$

Recall: u substitution lets us recognize the result of the chain rule. Integration by parts uses the product rule, but more subtly.

example

evaluate

$$\int x\, e^x \, \mathrm{d}x$$

Use u=x, dv=e^x dx.

When working this example, include constant of integration for v, show it cancels out, so we don't need to do that.

example

evaluate

$$\int x^2\, e^x \, \mathrm{d}x$$

Use the result of the previous example to solve this integral.

Moral: integration by parts may simplify things, but they may still need more simplification.

example

evaluate

$$\int x^5\, e^{x^2}\, \mathrm{d}x$$

Start with a u-substitution to simplify things: u=x^2. Then the problem essentially becomes the previous example.

Moral: sometimes combine techniques

This question from Leithold TC7, 7.1.1 6.

example

evaluate

$$\int \ln(x)\, \mathrm{d}x$$

Here we have no choice but to take dv=dx. It seems like this would make our integral more complicated, but it cancels with the derivative of ln.

example

evaluate

$$\int_1^{\sqrt{3}} \tan^{-1} x \, \mathrm{d}x$$

the antiderivative is

x atan(x)-1/2*ln(1+x^2)+C

atan(sqrt(3))=pi/3,
atan(1)=pi/4

Moral: works for definite integrals too

example

evaluate

$$\int e^x \sin x \, \mathrm{d}x$$

here we do integration by parts twice, add, divide by two. don't forget about the constant of integration.

Must use u=e^x twice, otherwise won't get anywhere.

trigonometric integrals

preliminary example

show

$$\int \tan x \, \mathrm{d}x = \ln |\sec x| + C$$

Write tan=sin/cos, do a u substitution.

preliminary example

show

$$\int \sec x \, \mathrm{d}x = \ln|\sec x + \tan x| + C$$

multiply top and bottom by (sec x + tan x), use

u=sec x + tan x

Know to do this because we want the answer to be ln (u), which comes from integrating du/u.

Note: Can divide third equation by cos^2 or sin^2 to get results about sec^2 or csc^2, respectively.

selected trig formulas

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin(\alpha)\,\cos(\beta)+\sin(\beta)\,\cos(\alpha)\\
\cos(\alpha+\beta) &= \cos(\alpha)\,\cos(\beta)-\sin(\alpha)\,\sin(\beta)\\
1 &= \sin^2(\alpha)+\cos^2(\alpha)\\
\sin(2\,\alpha) &= 2\,\sin(\alpha)\,\cos(\alpha)\\
\cos(2\,\alpha) &= \cos^2(\alpha)-\sin^2(\alpha)\\
\cos(2\,\alpha) &= 1-2\,\sin^2(x)\\
\cos(2\,\alpha) &= 2\,\cos^2(x)-1\\
\sin(\alpha)\,\cos(\beta) &= \left(\sin(\alpha-\beta)+\sin(\alpha+\beta)\right)/2\\
\sin(\alpha)\,\sin(\beta) &= \left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)/2\\
\cos(\alpha)\,\cos(\beta) &= \left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)/2
\end{aligned}$$

The bottom seven formulas follow from the top three.

Sometimes convenient to solve the formulas for cos(2a) for sin^2 or cos^2 instead. Then called half-angle formulas.

example

we know

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

but what about

$$\int \sin^2 x \, \mathrm{d}x$$

$$\int \sin^3 x \, \mathrm{d}x$$

Formulas for sin^2 and cos^2 follow from cos(2x) formula.

For sin^3, use pythagorean identity.

example

evaluate

$$\int \sin^3(2x)\,\cos^3(2x)\,\mathrm{d}x$$

Turn two of the cosines into sines, then

let u=sin(2x). Then du=2 cos(2x) dx

Alternatively could turn sines into cosines. Answer would look different, would need trig identities to show same.

$$\int \sin^m(x)\ \cos^n(x)\,\mathrm{d}x$$

May need to do a u-substitution first to get your expression into this form. In the last example, we could have made the substitution v=2x first to get it into form, then used these rules.

Note: The substitution will only work if sin and cos are both working on the same input... won't work with
sin(3x) cos(7x+2)

m odd	turn all but one sine into a cosine, then let $u = \cos(x)$
n odd	turn all but one cosine into a sine, then let $u = \sin(x)$
m and n even	use half-angle identities

example

evaluate

$$\int \csc^{20}(x)\ \cos^5(x)\,\mathrm{d}x$$

This may look different, but it's not, since csc^(20) (x)=sin^(-20) (x)

Four of the cosines becomes sines, u=sin(x), integrate, should get

u^(-19)/(-19)-2u^(-17)/(-17)+u^(-15)/(-15)+C

(plug back in u=sin(x), or better u^(-1)=csc(x))

example

evaluate

$$\int \tan^5(x)\ \sec^4(x)\,\mathrm{d}x$$

We can do this problem in two ways:

1. turn two secants into tan, (sec^2 x=1+tan^2 x), then u=tan x

or

2. turn 4 tangents into secants (tan^2 x = sec^2 x - 1), then u=sec x

The two options will look different, so may need trig identities to show the same.

$$\int \tan^m(x)\ \sec^n(x)\ \mathrm{d}x$$

m odd	turn all but one tangent into secant, then let $u = \sec(x)$
n even	turn all but **two** secant into tangent, then let $u = \tan(x)$
otherwise	no general rule

As before, here tangent and secant must both have the same input to use these rules.

example

evaluate

$$\int \sec^3(x)\,\mathrm{d}x$$

This is Stewart's example 8.

This doesn't fall into one of our rules. First do an integration by parts,

u=sec(x), dv=sec^2 (x) dx.

Turn the tan^2 into sec^2 - 1, solve for the integral of sec^3, use the formula for integral of sec discovered earlier to conclude

(sec x tan x + ln | sec x + tan x|)/2 + C

example

evaluate

$$\int \sin(3\,x)\ \sin(4\,x)\,\mathrm{d}x$$

Here we must use the product identities.

sin(3x) sin(4x) = cos(-x) - cos(7x)

integrating, we arrive at

sin(x) - 1/7 * sin(7x) + C

coming soon

- webwork project 1 (extra credit) due monday at 6 am
- homework 4 due tuesday
- read § 7.3 for tuesday
- quiz tuesday
- midterm on thursday

They know everything they need to know for project 2 and most of what they need for project 3. Project 2 due at 6am on 16 November.